

Not just the option to exclude: Valuing patents as a portfolio of options^{*}

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Abstract

We use a multi-stage real options model of duopoly to study the value of patents for innovative firms. Patents are often modelled as if they perfectly exclude competitors from market entry. However, in practice, exclusions only occur as a result of litigation, which is not necessarily successful. An alternative view is that patents automatically give rise to settlement and hence royalties. However, rivals only have the incentive to settle when facing a threat of litigation. We thus view patents as a portfolio of options, among which the most important for a producing firm is the option to litigate and the option to offer a settlement. We find that a rival entrant's gain in profits relative to the incumbent's resulting loss of profits ("gain-to-loss ratio") is a key determinant of patent values. Whilst the option value of litigation decreases with the gain-to-loss ratio, the option value of settlement increases with it, and for sufficiently high gain-to-loss ratios, when settlement is feasible, our patent value can exceed the value from simply excluding all rivals.

Keywords: *Patent value, litigation, settlement, real options*

JEL Classification: *O34; L10; D81; C73*

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1 The introduction

A patented technology derives value from two sources. The first is the *technological value*, reflecting profit flows generated by the underlying technology regardless of patent protection. The second, which is the focus of this paper, is the *patent value* arising from the exclusive rights conferred by the patent. These rights allow the patent owner to use enforcement mechanisms to protect profits when infringement occurs. We propose a novel decomposition of patent value into two components: *litigation value*, associated with the option to pursue legal action against infringers, and *settlement value*, associated with the option to resolve disputes through negotiated settlement agreements.

Since the 1980s, shifts in US patent policy have renewed the question of whether stronger patent protection promotes or hinders innovation. On one side, the pro-innovation view argues that expanding intellectual property (IP) rights strengthens incentives for research and development (R&D) by granting temporary monopoly rents, while on the other side, critics contend that excessive patenting, patent thickets, and escalating litigation costs distort competition and discourage follow-on research (National Research Council, 2003; Shapiro, 2000; Hall, Jaffe, and Trajtenberg, 2000). A key missing piece of the debate lies in understanding the drivers of the value of patented technology, which reflects both the technological quality of an invention (*technology value*) and the legal strength of the associated rights (*patent value*).

Using a parsimonious duopoly model with stochastic market demand and imperfect patent protection, we derive closed-form solutions for patent value. This framework allows us to decompose the total patent value into two distinct components: a litigation option value and a settlement option value. These analytical solutions link patent value to several key drivers, including the degree of patent protection, demand growth and volatility, the business-stealing effect of entry, and the profitability of new entrants. Our findings indicate that while stronger patent protection increases both the total patent value and its litigation option value, it may counterintuitively decrease the value of the settlement option. This occurs if the time value of the additional delay in the challenger's entry, caused by stronger protection, outweighs the benefit of the increased expected settlement payoff.

The model highlights how product market characteristics fundamentally shape patent value. A key determinant in this framework is the *gain-to-loss ratio*, defined as the challenger's profit gain relative to the incumbent's loss, both resulting from the challenger's market entry. We show that while a high ratio makes settlement feasible, it reduces the total patent value by lowering the stakes of potential litigation. Crucially, we find that when settlement is viable, the total patent value can exceed the value of guaranteeing a monopoly by excluding all rivals. This mechanism complements existing work on litigation and settlement (Marco, 2005; Jeon, 2015) and aligns with broader theories of dispute resolution under asymmetric incentives (Bebchuk, 1984; Spier, 2007; Liu and Lin, 2026).

This paper contributes to the intersection of economics, law, and finance literature. We offer one of the first theoretical decompositions of patent value into litigation and settlement components, establishing a unified framework linking patent enforceability to innovation incentives. We also extend the real options approach to capture the sequential and strategic nature of patent enforcement, complementing prior studies on litigation (Marco, 2005; Jeon, 2015).

2 Model

In our model, an incumbent (“I”) holds a patent and earns a monopoly profit flow $\pi_1 x_t dt$ from the patented technology. When a challenger (“C”) enters, paying a fixed cost I^C and using a similar technology, the two firms earn duopoly profit flows $\pi_2^I x_t dt$ and $\pi_2^C x_t dt$. Entry reduces the incumbent’s profit, so $\pi_2^I < \pi_1$. The stochastic market demand x_t follows a geometric Brownian motion (GBM):

$$dx_t = \mu x_t dt + \sigma x_t dW_t, \quad (1)$$

where the demand growth rate satisfies $\mu < r$ (the risk free rate), $\sigma > 0$, and W_t is a standard Brownian motion.

Based on the changes in the linear profit parameters triggered by the entry of the challenger, we define:

$$\text{Incumbent's loss-of-profit: } \Delta\pi^I \equiv |\pi_2^I - \pi_1| = \pi_1 - \pi_2^I > 0 \quad (2)$$

$$\text{Challenger's gain-of-profit: } \Delta\pi^C \equiv |\pi_2^C - 0| = \pi_2^C > 0 \quad (3)$$

$$\text{The gain-to-loss ratio: } \Phi \equiv \frac{\Delta\pi^C}{\Delta\pi^I} = \frac{\pi_2^C}{\pi_1 - \pi_2^I} \quad (4)$$

$$\text{The market size change: } \Delta\pi = \pi_2 - \pi_1 = (\pi_2^I + \pi_2^C) - \pi_1 = \Delta\pi^C - \Delta\pi^I \leq 0 \quad (5)$$

The gain-to-loss ratio quantifies the relative impact of a challenger’s entry by comparing their gains against the incumbent’s losses. The numerator represents the profitability of the new entrant, while the denominator captures the “business stealing effect” where the incumbent’s value is eroded. By focusing on the profit shift between competitors, this ratio highlights the direct impact on the firms involved rather than the broader changes to consumer surplus resulting from the transition from monopoly to duopoly.

After the challenger’s entry, the incumbent can sue the challenger for infringement. With probability p , the incumbent wins the case and restores its monopoly profits; with probability $1 - p$, the challenger wins, and the two firms continue to share the market with the status quo. Alternatively, they may settle through a royalty payment, which is a fixed proportion of the challenger’s profits paid to the incumbent in all future periods.

Firms follow threshold strategies in optimality: an action is taken once the market demand x_t reaches a critical

level. To deliver the key messages of the model on patent value decomposition, we assume the court decision is immediate, litigation and settlement costs are negligible, no damages are awarded, and the patent does not expire. Under these assumptions, it is optimal for the incumbent to litigate as soon as the challenger enters.

2.1 The Patent Value without a Settlement Option

Challenger Entry Given the common knowledge of the litigation win probability p , the challenger's entry decision is characterized by the following value maximization problem:

$$V_p^C(x) = \max_{\tau} \mathbb{E} \left[\int_{\tau}^{\infty} e^{-rt} (1-p) \pi_2^C dt - e^{-\tau t} I^C \mid x_0 = x \right] \quad (6)$$

Before entry, the challenger's value $V_p^C(x)$ satisfies the Bellman equation

$$rV_p^C(x) dt = \mathbb{E} dV_p^C(x), \quad \mathbb{E} dV_p^C(x) = \left(\frac{1}{2} \sigma^2 x^2 V_{xx}^C + \mu x V_x^C \right) dt.$$

Its solution takes the form

$$V_p^C(x) = Ax^{\beta_1}, \quad \beta_1 = \frac{1}{2} - \frac{\mu}{\sigma^2} + \sqrt{\left(\frac{1}{2} - \frac{\mu}{\sigma^2} \right)^2 + \frac{2r}{\sigma^2}} > 1. \quad (7)$$

Under standard assumptions, optimal entry timing reduces to choosing the threshold $\bar{x}_p$ (with p denoting the incumbent's litigation win probability) above which the challenger enters. Applying value matching and smooth pasting yields the optimal threshold and the constant A :

$$\bar{x}_p = \frac{\beta_1}{\beta_1 - 1} \frac{I^C(r - \mu)}{(1-p)\pi_2^C} = \frac{\beta_1}{\beta_1 - 1} \frac{I^C(r - \mu)}{(1-p)\Delta\pi^C}, \quad A = \frac{I^C \bar{x}_p^{-\beta_1}}{\beta_1 - 1}. \quad (8)$$

With perfect patent protection ($p = 1$), the challenger never enters because litigation would immediately force it out of the market. With no patent protection ($p = 0$), the entry threshold reduces to the standard follower rule in a duopoly (e.g., [Kong and Kwok, 2007](#)), resulting in a more aggressive entry than under any positive level of patent protection ($p > 0$):

$$\bar{x}_0 = \frac{\beta_1}{\beta_1 - 1} \frac{I^C(r - \mu)}{\Delta\pi^C} < \bar{x}_p. \quad (9)$$

Lemma 1. *With patent protection level p and no settlement option, the challenger's value function is*

$$V_p^C(x) = \begin{cases} \frac{I^C}{\beta_1 - 1} \left(\frac{x}{\bar{x}_p} \right)^{\beta_1}, & x < \bar{x}_p, \\ \frac{(1-p)\pi_2^C x}{r - \mu}, & x \geq \bar{x}_p. \end{cases}$$

The optimal entry threshold $\bar{x}_p$ is increasing in patent protection and demand uncertainty ($\frac{\partial \bar{x}_p}{\partial p} > 0$ and $\frac{\partial \bar{x}_p}{\partial \sigma^2} > 0$).

Technology Value versus Litigation Value Before the challenger's entry, the incumbent's firm value V_p^I satisfies

$$rV_p^I(x)dt = \mathbb{E} dV_p^I + \pi_1 x dt. \quad (10)$$

Its general solution is $V_p^I(x) = ax^{\beta_1} + \frac{\pi_1}{r-\mu}x$, with a determined by value matching. With litigation but no settlement option, the incumbent's value is:

$$V_p^I(x) = \begin{cases} \frac{\pi_1}{r-\mu}x - \frac{\beta_1}{\beta_1-1} \frac{I^C \Delta \pi^I}{\Delta \pi^C} \left(\frac{x}{\bar{x}_p}\right)^{\beta_1}, & \text{if } x < \bar{x}_p \\ \frac{\pi_2 + p \Delta \pi^I}{r-\mu}x, & \text{if } x \geq \bar{x}_p. \end{cases} \quad (11)$$

In the special case of $p = 0$, the incumbent has no meaningful litigation option, and its value reflects only the payoff from the underlying technology, regardless of whether the technology is patented. We refer to this as the *value of the technology*.

Lemma 2. *The incumbent's technology value is*

$$V_{tech}^I(x) = V_{p=0}^I = \begin{cases} \frac{\pi_1}{r-\mu}x - \frac{\beta_1}{\beta_1-1} \frac{I^C \Delta \pi^I}{\Delta \pi^C} \left(\frac{x}{\bar{x}_0}\right)^{\beta_1}, & \text{if } x < \bar{x}_0 \\ \frac{\pi_2^I}{r-\mu}x, & \text{if } x \geq \bar{x}_0 \end{cases} \quad (12)$$

where $\bar{x}_0$ is the challenger's entry threshold in the absence of the incumbent's patent protection, given in Eq (9).

Conceptually, without a settlement option, the *patent value from litigation* is the difference between the incumbent's value with the litigation option (V_p^I) and its technology value (V_{tech}^I).

Lemma 3. *The patent value from its litigation option is*

$$V_{litg}(x) \equiv V_p^I(x) - V_{tech}^I(x) = \begin{cases} \frac{\bar{x}_p(1-p)\Delta\pi^C}{(r-\mu)\Phi} \left[\left(\frac{x}{\bar{x}_0}\right)^{\beta_1} - \left(\frac{x}{\bar{x}_p}\right)^{\beta_1} \right] & \text{if } x < \bar{x}_0 \text{ with } \bar{x}_0 \text{ specified in Eq.(9)} \\ \frac{\bar{x}_p\Delta\pi^C}{(r-\mu)\Phi} \left[\frac{x}{\bar{x}_p} - (1-p)\left(\frac{x}{\bar{x}_p}\right)^{\beta_1} \right] & \text{if } \bar{x}_0 \leq x < \bar{x}_p \\ \frac{p\Delta\pi^C}{(r-\mu)\Phi}x & \text{if } x \geq \bar{x}_p \text{ with } \bar{x}_p \text{ specified in Eq.(8)} \end{cases} \quad (13)$$

2.2 Patent Value Decomposition: Litigation and Settlement Options

We model settlement as the incumbent proposing a royalty rate θ and the challenger choosing whether and when to accept it (Lukas, Reuer, and Welling, 2012). Under settlement, the challenger pays a fraction θ of its future profits to the incumbent. Since settlement and litigation are costless in our setting, any settlement occurs immediately at the challenger's market entry $\bar{x}_p$. Both parties compare settlement to their expected payoffs from litigation: the incumbent settles only if the royalty at least matches its expected profit regain under litigation, and the challenger accepts to settle only if it does not exceed its expected loss in litigation. These conditions are:

$$\text{Incumbent prefers to settle if: } \frac{(\pi_2^I + \theta\pi_2^C)x}{r - \mu} \geq \frac{\pi_2^I + p(\pi_1 - \pi_2^I)}{r - \mu}x \Rightarrow \theta \geq \frac{p}{\Phi} \equiv \theta^{Imin} \quad (14)$$

$$\text{Challenger prefers to settle if: } \frac{\pi_2^C(1 - \theta)}{r - \mu}x \geq \frac{\pi_2^C(1 - p)}{r - \mu}x \Rightarrow \theta \leq p \equiv \theta^{Cmax} \quad (15)$$

Settlement occurs only if the maximum royalty the challenger is willing to pay is at least as large as the minimum the incumbent will accept, i.e., $\theta^{Cmax} \geq \theta^{Imin}$. This strategic compatibility condition reduces to a requirement on the gain-to-loss ratio, formalized in Lemma 4.

Lemma 4. *Settlement is feasible if and only if the gain-to-loss ratio $\Phi \geq 1$. The agreed-upon royalty rate equals the incumbent's probability of winning in litigation, $\theta = p$.*

When settlement is feasible ($\Phi \geq 1$), the incumbent optimally proposes the maximum royalty the challenger accepts, $\theta = \theta^{Cmax} = p$.¹ Settlement then occurs immediately at $x = \bar{x}_p$. The value-matching condition implies:

$$\begin{aligned} a_s \bar{x}_p^{\beta_1} + \frac{\pi_1}{r - \mu} \bar{x}_p &= \frac{\theta\pi_2^C + \pi_2^I}{r - \mu} \bar{x}_p = \frac{p\pi_2^C + \pi_2^I}{r - \mu} \bar{x}_p \\ \Rightarrow a_s &= -\bar{x}_p^{-\beta_1} \frac{\beta_1}{\beta_1 - 1} \frac{I^C}{1 - p} \left(\frac{1}{\Phi} - p \right) \end{aligned} \quad (16)$$

The incumbent's value with both litigation and settlement options, before the challenger's entry is:

$$V_{s,p}^I(x) = \begin{cases} \frac{\pi_1 x}{r - \mu} - \frac{\beta_1}{\beta_1 - 1} I^C \frac{\frac{1}{\Phi} - p}{1 - p} \left(\frac{x}{\bar{x}_p} \right)^{\beta_1}, & \text{if } x < \bar{x}_p \\ \frac{\pi_2^I + p\pi_2^C}{r - \mu} x, & \text{if } x \geq \bar{x}_p \text{ with } \bar{x}_p \text{ specified in Eq.(8)} \end{cases} \quad (17)$$

We argue that the total patent value is the difference between the incumbent's value with both litigation and settlement options and its technology value, that is, $V_{\text{patent}} = V_{s,p}^I(x) - V_{\text{tech}}^I(x)$. The settlement component of the patent value is the difference between the incumbent's value with both options and its value with litigation

¹When settlement is feasible, the incumbent effectively holds full bargaining power: settlement occurs at entry, the incumbent proposes the rate, and the challenger chooses whether to accept.

option only, that is, $V_{\text{patent}}^I - V_{\text{ltg}}^I$. This option to settle also implies that the total patent value can be more than the value of guaranteeing a monopoly.²

Lemma 5. *The total patent value $V_{\text{patent}}(x)$ decomposes into the litigation option component $V_{\text{ltg}}(x)$ in Eq. (13) and the settlement option component $V_{\text{settlement}}$. When settlement is feasible ($\Phi \geq 1$):*

$$V_{\text{patent}}(x) = V_{s,p}^I - V_0^I = \begin{cases} \frac{\bar{x}_p \Delta \pi^C}{(r - \mu) \Phi} \left[(1 - p) \left(\frac{x}{\bar{x}_0} \right)^{\beta_1} - (1 - p\Phi) \left(\frac{x}{\bar{x}_p} \right)^{\beta_1} \right], & x < \bar{x}_0, \\ \frac{\Delta \pi^C}{(r - \mu) \Phi} \left[x - \bar{x}_p (1 - p\Phi) \left(\frac{x}{\bar{x}_p} \right)^{\beta_1} \right], & \bar{x}_0 \leq x < \bar{x}_p, \\ \frac{p \Delta \pi^C}{r - \mu} x, & x \geq \bar{x}_p. \end{cases} \quad (18)$$

$$V_{\text{settlement}} \equiv V_{\text{patent}} - V_{\text{ltg}} = \begin{cases} \frac{p \Delta \pi}{r - \mu} \left(\frac{x}{\bar{x}_p} \right)^{\beta_1 - 1} x, & x < \bar{x}_p, \\ \frac{p \Delta \pi}{r - \mu} x, & x \geq \bar{x}_p. \end{cases} \quad (19)$$

When settlement is not feasible ($\Phi < 1$), $V_{\text{settlement}} = 0$ and $V_{\text{patent}} = V_{\text{ltg}}$.

Figure 1 decomposes the incumbent's value under imperfect patent protection ($p = 0.5$) when settlement is feasible ($\Phi = 1.5$), evaluated at the baseline parameters in Table 1. Total value consists of technology value (V_{tech}^I , lined shaded area) and patent value, with the latter further decomposed into litigation (V_{ltg} , dark solid shaded area) and settlement ($V_{\text{settlement}}$, light solid shaded area) components. While both technology value and total patent value increase with market demand, the linearity of technology value depends on the challenger's entry decision, and the litigation value is non-monotonic in market demand.³

[insert Figure 1 here]

The following result shows how the patent value V_{patent} and its components (V_{ltg} and $V_{\text{settlement}}$) vary with the gain-to-loss ratio and the strength of patent protection.⁴

Proposition 1. $\frac{\partial V_{\text{patent}}}{\partial \Phi} \leq 0$, $\frac{\partial V_{\text{ltg}}}{\partial \Phi} \leq 0$, $\frac{\partial V_{\text{settlement}}}{\partial \Phi} \geq 0$; $\frac{\partial V_{\text{patent}}}{\partial p} > 0$, $\frac{\partial V_{\text{ltg}}}{\partial p} > 0$, $\frac{\partial V_{\text{settlement}}}{\partial p} \geq 0$.

The value of the litigation option decreases as the gain-to-loss ratio, Φ , increases. A higher Φ implies a smaller profit loss for the incumbent, which directly reduces the incentive to litigate. In contrast, the settlement value increases with Φ for two reasons: first, settlement only becomes feasible once Φ exceeds one (as shown in Lemma 4); and second, a lower profit loss for the incumbent improves the available settlement terms.

²From Equation (18) in Lemma 5, for $x \geq \bar{x}_p$, if $\Phi > \frac{1}{p}$, then $V_{\text{patent}} > \frac{\Delta \pi^I}{r - \mu} x$.

³The thin red line shows technology value only, the thicker blue line adds the litigation option ($p = 0.5$), and the thickest green line includes both litigation and settlement options.

⁴For tractability, when studying the effect of Φ , we hold π_2^C and π_1 fixed and vary π_2^I , which keeps the entry threshold $\bar{x}_p$ unchanged.

Despite this, Proposition 1 demonstrates that the positive effect on settlement is insufficient to offset the decline in litigation value; consequently, the total patent value falls as Φ increases. Meanwhile, stronger patent protection (p) enhances both total patent value and the value of the litigation option. However, this relationship is not universal for settlement: when $p\beta_1 > 1$, the settlement value actually decreases with p prior to the challenger's entry. This represents the impact of two opposing forces: as p increases, the incumbent's value on settlement increases but it has to endure a longer wait before settlement occurs due to the delayed entry of the challenger. The incumbent's settlement option value decreases when p increases if the time value of the additional delay of the challenger's entry outweighs the impact of the increase of the expected settlement payoff.

Figure 2 illustrates the baseline decomposition of total patent value relative to the gain-to-loss ratio, Φ , across three distinct market states: before challenger's entry with or without patent protection, before entry with protection and after entry without, and after entry regardless of protection status. When $\Phi < 1$, settlement remains infeasible (as established in Lemma 4), meaning the total patent value consists entirely of the litigation value, both of which decline as Φ increases. Once Φ exceeds the threshold of 1, settlement becomes viable, introducing settlement value (the shaded area) as a new component of the total patent value that grows as Φ continues to increase.

[insert Figure 2 here]

Figure 3 illustrates the decomposition of patent value relative to market demand across three levels of patent protection (p). The blue lines represent total patent value, the red lines represent litigation value, and the shaded areas denote settlement value, all calculated at a baseline of $\Phi = 1.5$ to ensure settlement feasibility. The line thickness corresponds to the strength of protection: the thinnest lines represent $p = 0.25$, the medium lines represent $p = 0.5$, and the thickest lines represent $p = 0.75$. Several key insights emerge from the figure: first, patent value does not always increase with market demand. Specifically, patent value may decrease as demand (x) grows when patent protection (p) is low and x falls within a middle range where the challenger would have entered the market in the absence of protection but is currently deterred. Second, consistent with the findings in Proposition 1, while an increase in p increases both the total patent value and the litigation value component, it may simultaneously cause the settlement value to decrease. This emphasizes that the balance between β_1 and $1/p$ in the model is crucial for patent value.

[insert Figure 3 here]

3 Conclusion

Patent valuation remains a complex challenge because traditional cost, revenue, and market approaches often fail to account for uncertain enforcement and dynamic market conditions. This makes real options methods particu-

larly relevant. This paper develops a theoretical model that incorporates uncertainty in both market demand and the patent enforcement process, allowing us to distinguish the intrinsic value of the technology from the strategic value provided by the patent itself.

Crucially, we demonstrate that the value of a patent is distinct from the value generated by simply excluding rivals. Given that patent protection is often imperfect, the total patent value is comprised of both the option value to litigate and, when settlement is feasible, the additional option value of reaching a settlement. This additional value of settling implies that the total patent value can be more than the value of guaranteeing a monopoly. We show that the gain-to-loss ratio between the alleged infringer and the patent holder plays a central role in this decomposition: while a higher ratio increases the settlement component, it simultaneously reduces the litigation component, leading to an overall decline in total patent value.

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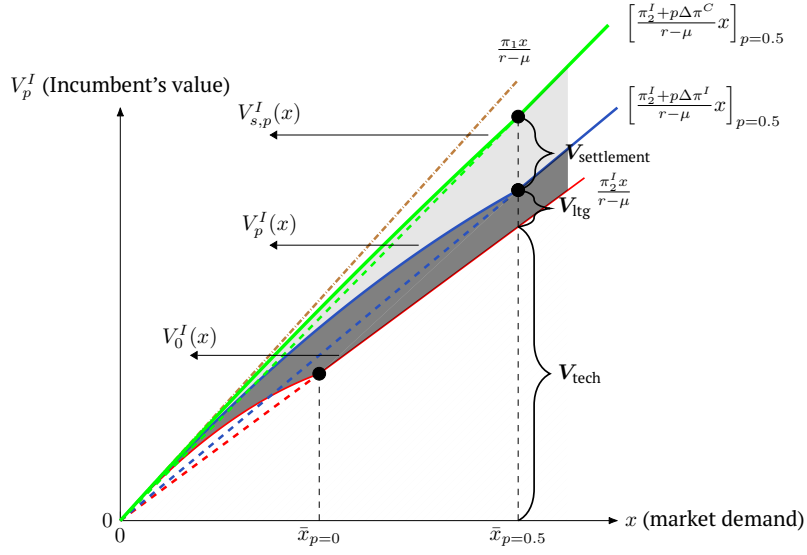


Figure 1: Incumbent value decomposition for $p = 0.5$

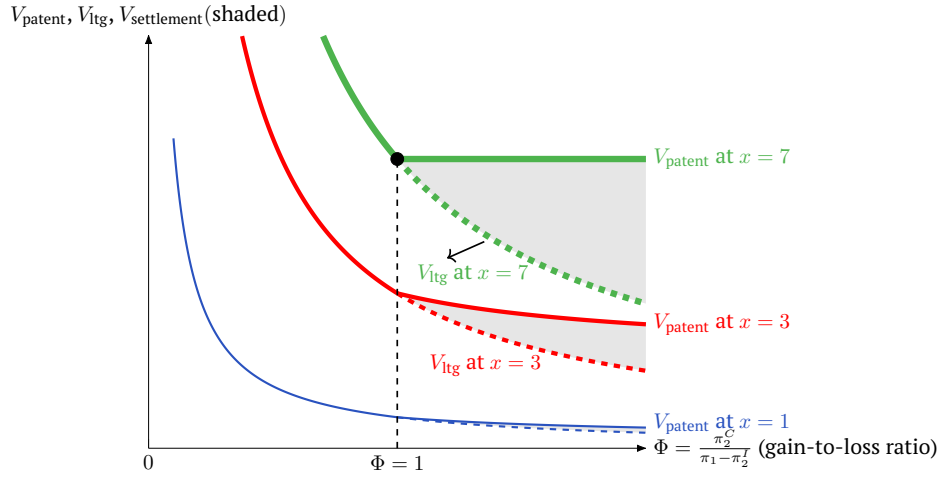


Figure 2: Patent value and its decomposition vs. Φ at different demand x

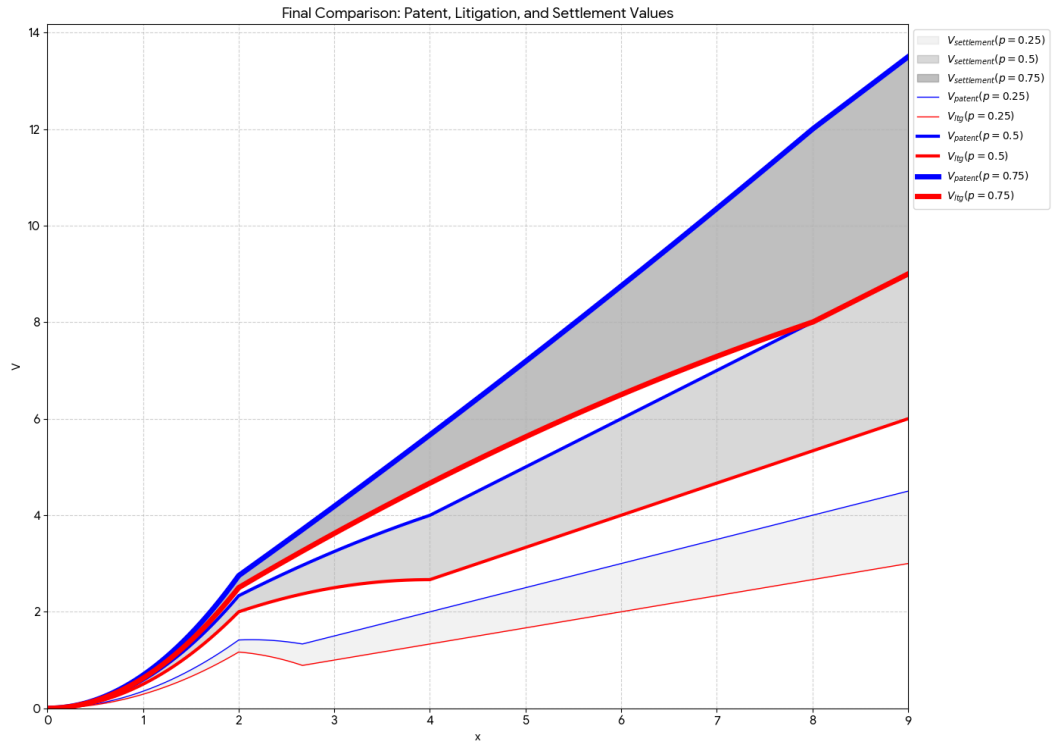


Figure 3: Patent value and its decomposition vs. x at three different values of p

Table 1: Baseline parameter values

Parameter	Value
Risk free rate	$r = 0.05$
Probability of patent validity	$p = 0.5$
Growth rate/volatility of the demand shock	$\mu = 0.02, \sigma = 0.1$
I's monopoly profit multiplier (profit = $\pi_1 x$)	$\pi_1 = 0.12$
Duopoly profit multipliers	$\pi_2^I = 0.08, \pi_2^C = 0.06$
Entry investment cost for the challenger	$I^C = 2$
Incumbent's loss-of-profit	$\Delta\pi^I = \pi_1 - \pi_2^I = 0.04$
<u>Ratios</u>	
gain-to-loss ratio	$\Phi = \frac{\pi_2^C}{\pi_1 - \pi_2^I} = 1.5$

A Proofs and derivations

A.1 Proof of Proposition 1

Proof. From Eq(19),

$$\text{For } \Phi \geq 1 : \frac{\partial V_{\text{settlement}}}{\partial \Phi} \begin{cases} = \frac{p\pi_2^C}{r-\mu} \frac{x}{\Phi^2} > 0, & \text{if } x \geq \bar{x}_p \\ = \frac{p\pi_2^C}{r-\mu} \frac{1}{\Phi^2} \times \left(\frac{x}{\bar{x}_p}\right)^{\beta_1-1} > 0. & \text{if } x \leq \bar{x}_p, \text{ and } \frac{\partial \bar{x}_p}{\partial \Phi} = 0 \end{cases} \quad (20)$$

$$\text{For } \Phi < 1 : \frac{\partial V_{\text{settlement}}}{\partial \Phi} = 0. \quad (21)$$

In addition, $V_{\text{settlement}}(\Phi > 1) > V_{\text{settlement}}(\Phi \leq 1)$. Together: $\frac{\partial V_{\text{settlement}}}{\partial \Phi} \geq 0$ for all $\Phi > 0$.

From Eq(19),

$$\text{For } \Phi \geq 1 : \frac{\partial V_{\text{settlement}}}{\partial p} \begin{cases} = \textcircled{1} = \frac{\pi_2^C}{r-\mu} x \left(1 - \frac{1}{\Phi}\right) > 0, & \text{if } x \geq \bar{x}_p \\ = \textcircled{2} = \frac{\pi_2^C}{r-\mu} x \left(1 - \frac{1}{\Phi}\right) \left(\frac{x}{\bar{x}_p}\right)^{\beta_1-1} \frac{1-p\beta_1}{1-p}, & \text{if } x \leq \bar{x}_p. \end{cases} \quad (22)$$

Given $\beta_1 > 1$, we have $\frac{\textcircled{2}}{\textcircled{1}} = \left(\frac{x}{\bar{x}_p}\right)^{\beta_1-1} \frac{1-p\beta_1}{1-p} < 1$ in Region 2, i.e., $\textcircled{2} < \textcircled{1}$. Further, $\frac{\partial \bar{x}_p}{\partial p} > 0$, which indicates a shift of some high-demand region for a low-p to mid-demand region for a high-p. However, when $p\beta_1 > 1$, then $\textcircled{2} < 0$.

Based on Eq(18), for $\Phi \geq 1$:

$$\frac{\partial V_{\text{patent}}}{\partial \Phi} \begin{cases} = 0, & \text{if } x \geq \bar{x}_p \\ = \textcircled{2} = \frac{\pi_2^C}{(r-\mu)\Phi^2} \left[-1 + \left(\frac{x}{\bar{x}_p}\right)^{\beta_1-1} \right] < 0, & \text{if } \bar{x}_0 \leq x \leq \bar{x}_p \\ = \textcircled{3} = \frac{\bar{x}_p \pi_2^C}{(r-\mu)\Phi^2} \left[-(1-p)\left(\frac{x}{x_0}\right)^{\beta_1} + \left(\frac{x}{\bar{x}_p}\right)^{\beta_1} \right], & \text{if } x \leq \bar{x}_0. \end{cases} \quad (23)$$

$\text{sign}\left(\frac{\partial \textcircled{3}}{\partial x}\right) = \text{sign}(-(1-p) + (1-p)^{\beta_1}) = \text{negative}$. $\textcircled{3}(\bar{x}_0) = -(1-p) + (1-p)^{\beta_1} < 0$, and $\lim_{x \rightarrow 0} \textcircled{3} = 0$, thus $\textcircled{3} \leq 0$.

Take the first order derivative of Eq (13) with respect to p in each region, we get

$$\frac{\partial V_{\text{ltg}}}{\partial p} = \begin{cases} \frac{\Delta\pi^I}{r-\mu}x > 0 & \text{if } x \geq \bar{x}_p, \\ \frac{\partial V_{\text{ltg}}}{\partial \bar{x}_p} \times \frac{\partial \bar{x}_p}{\partial p} = \frac{\beta_1(1-p)\Delta\pi^I}{r-\mu} \left(\frac{x}{\bar{x}_p}\right)^{\beta_1} \times \frac{\partial \bar{x}_p}{\partial p} = \frac{\beta_1\bar{x}_p\Delta\pi^I}{r-\mu} \left(\frac{x}{\bar{x}_p}\right)^{\beta_1} > 0 & \text{if } \bar{x}_0 \leq x \leq \bar{x}_p, \\ \frac{\partial V_{\text{ltg}}}{\partial \bar{x}_p} \times \frac{\partial \bar{x}_p}{\partial p} = \frac{\beta_1\bar{x}_p\Delta\pi^I}{r-\mu} \left(\frac{x}{\bar{x}_p}\right)^{\beta_1} > 0 & \text{if } x \leq \bar{x}_0. \end{cases} \quad (24)$$

V_{ltg} is continuous in p , thus $\frac{\partial V_{\text{ltg}}}{\partial p} > 0$.

From Eq(13),

$$\frac{\partial V_{\text{ltg}}}{\partial \Phi} \begin{cases} < 0, & \text{if } x \geq \bar{x}_p \\ < 0, & \text{if } \bar{x}_0 \leq x \leq \bar{x}_p \\ < 0, & \text{if } x \leq \bar{x}_0. \end{cases} \quad (25)$$

Given $\frac{\partial \bar{x}_0}{\partial \Phi} = 0$ and $\frac{\partial \bar{x}_p}{\partial \Phi} = 0$, $\frac{\partial V_{\text{ltg}}}{\partial \Phi} < 0$ for all Φ . □